

## Solving Radical Equations - Solutions

## Solve radical equations and check for extraneous solution

**We Do #1:** Solve

$$\begin{aligned}\sqrt{x} &= 5 \\ (\sqrt{x})^2 &= (5)^2 \\ x &= 25\end{aligned}$$

Check it:

$$\begin{aligned}\sqrt{x} &= 5 \\ \sqrt{25} &= 5 \\ 5 &= 5\end{aligned}$$

What value makes this sentence true?

$\therefore x = 25$  is a solution.

**We Do #2:**

$$\begin{aligned}\sqrt{x} &= -7 \\ (\sqrt{x})^2 &= (-7)^2 \\ x &= 49\end{aligned}$$

Check it:

$$\begin{aligned}\sqrt{49} &= -7 \\ 7 &= -7\end{aligned}$$

False, so  $x = 49$  is not a solution.

**We Do #3:** Solve

$$\begin{aligned}\sqrt{x+2} &= 3 \\ (\sqrt{x+2})^2 &= (3)^2 \\ x+2 &= 9 \\ x &= 7\end{aligned}$$

Check it:

$$\begin{aligned}\sqrt{(7)+2} &= 3 \\ \sqrt{9} &= 3 \\ 3 &= 3 \text{ True} \\ \therefore x &= 7\end{aligned}$$

**You Try #1:**

$$\begin{aligned}\sqrt{x-9} &= 4 \\ (\sqrt{x-9})^2 &= (4)^2 \\ x-9 &= 16 \\ x &= 25\end{aligned}$$

Check it:

$$\begin{aligned}\sqrt{(25)-9} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \text{ True} \\ \therefore x &= 25\end{aligned}$$

**We Do #4:**

$$\begin{aligned}\sqrt{x-1} &= x-7 \\ (\sqrt{x-1})^2 &= (x-7)^2 \\ x-1 &= (x-7)(x-7) \\ x-1 &= x^2 - 14x + 49 \\ 0 &= x^2 - 15x + 50 \\ 0 &= (x-10)(x-5) \\ x &= 10 \text{ and } x = 5\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{(10)-1} &= (10)-7 & \sqrt{(5)-1} &= (5)-7 \\ \sqrt{9} &= 3 & \sqrt{4} &= -2 \\ 3 &= 3 \text{ True} & 2 &= -2 \text{ False}\end{aligned}$$

$\therefore x = 10$  is a solution.

There is an extraneous root at  $x = 5$

**You Try #2:**

$$\begin{aligned}\sqrt{x+1} &= x-1 \\ (\sqrt{x+1})^2 &= (x-1)^2 \\ x+1 &= (x-1)(x-1) \\ x+1 &= x^2 - 2x + 1 \\ 0 &= x^2 - 3x \\ 0 &= x(x-3) \\ x &= 0 \text{ and } x = 3\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{(0)+1} &= (0)-1 & \sqrt{(3)+1} &= (3)-1 \\ \sqrt{1} &= -1 & \sqrt{4} &= 2 \\ 1 &= -1 \text{ False} & 2 &= 2 \text{ True}\end{aligned}$$

$\therefore x = 3$  is a solution.

There is an extraneous root at  $x = 0$

**Teamwork:**

1.

$$\begin{aligned}\sqrt{n+9} &= 2 \\ (\sqrt{n+9})^2 &= (2)^2 \\ n+9 &= 4 \\ n &= -5\end{aligned}$$

$\therefore n = -5$  is a solution.

Check:

$$\begin{aligned}\sqrt{(-5)+9} &= 2 \\ \sqrt{4} &= 2 \\ 2 &= 2 \text{ True}\end{aligned}$$

2.

$$3 = \sqrt{4m}$$

$$(3)^2 = (\sqrt{4m})^2$$

$$9 = 4m$$

$$\frac{9}{4} = \frac{4m}{4}$$

$$\frac{9}{4} = m$$

$\therefore m = \frac{9}{4}$  is a solution.

Check:

$$3 = \sqrt{4\left(\frac{9}{4}\right)}$$

$$3 = \sqrt{\frac{36}{4}}$$

$$3 = \sqrt{9}$$

$$3 = 3 \text{ True}$$

3.

$$\sqrt{12-r} = r$$

$$(\sqrt{12-r})^2 = (r)^2$$

$$12-r = r^2$$

$$0 = r^2 + r - 12$$

$$0 = (x+4)(x-3)$$

$$x = -4 \text{ and } x = 3$$

Check:

$$\sqrt{12-(-4)} = (-4) \quad \sqrt{12-(3)} = (3)$$

$$\sqrt{16} = -4 \quad \sqrt{9} = 3$$

$$4 = -4 \text{ False} \quad 3 = 3 \text{ True}$$

$\therefore x = 3$  is a solution.

There is an extraneous root at  $x = -4$

4.

$$w = \sqrt{-4+4w}$$

$$(w)^2 = (\sqrt{-4+4w})^2$$

$$w^2 = -4+4w$$

$$w^2 - 4w + 4 = 0$$

$$(w-2)(w-2) = 0$$

$$w = 2$$

Check:

$$(2) = \sqrt{-4+4(2)}$$

$$2 = \sqrt{-4+8}$$

$$2 = \sqrt{4}$$

$$2 = 2 \text{ True}$$

$\therefore x = 2$  is a solution.

5.

Hint: Subtract 5 from both sides to isolate the radical.

$$\sqrt{x-4} + 5 = 12$$

$$\sqrt{x-4} = 7$$

$$(\sqrt{x-4})^2 = (7)^2$$

$$x-4 = 49$$

$$x = 53$$

Check:

$$\sqrt{(53)-4} + 5 = 12$$

$$\sqrt{49} + 5 = 12$$

$$7 + 5 = 12$$

$$12 = 12 \text{ True}$$

$\therefore x = 53$  is a solution.

6. Hint: Cube each side (raise to the third power) to undo the cube root.

$$\sqrt[3]{x-3} = 2$$

$$(\sqrt[3]{x-3})^3 = (2)^3$$

$$x-3 = 8$$

$$x = 11$$

Check:

$$\sqrt[3]{(11)-3} = 2$$

$$\sqrt[3]{8} = 2$$

$$2 = 2 \text{ True}$$

$\therefore x = 11$  is a solution.